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THE SATELLITE 1962 β α

by

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27260 ABSTRACT

A study of the satellite 1962 β_a (Alouette) during the period April 27, 1963 to March 8, 1964 revealed a long-period perturbation (approximately 140 days, the period of the argument of perigee) in the eccentricity of its orbit, plus shorter-period variations. An analysis of the possible sources of these perturbations was undertaken, neglecting air drag since perigee was about 1000 km.

Mean orbital elements for this period (Table 1), computed using Brouwer's satellite theory (1), were obtained from the Goddard computing center. These elements take into account the effects due to zonal harmonics (through J_5) in the earth's gravitational potential. Figure 1 illustrates the variation of e with time.

AUTHOR

LUNAR AND SOLAR GRAVITATIONAL EFFECTS

The disturbing function for both the sun and moon may be written in the form

$$R = \frac{Gm'}{r'} \left(\frac{1}{r'} + \frac{r^2}{r'^2} S_2 + \frac{r^3}{r'^3} S_3 + \dots \right),$$

where

G = gravitational constant

r = geocentric distance of satellite

r' = geocentric distance of sun (or moon)

S_2, S_3, \dots = Legendre polynomials

$$m' = \frac{\text{mass of disturbing body}}{\text{mass of disturbing body} + \text{mass of earth}} .$$

The variation equation for eccentricity is

$$\frac{de}{dt} = -\frac{\sqrt{1-e^2}}{na^2e} \frac{\partial R}{\partial \omega} + \frac{1-e^2}{na^2e} \frac{\partial R}{\partial M}$$

where

n = mean motion of satellite

a = semi-major axis of satellite's orbit

ω = argument of perigee

M = mean anomaly of satellite.

Assuming mean values for the elements and considering only the long-period terms from the second-order part of the disturbing function [Kozai (2)], a first approximation to δe reveals no terms with amplitudes greater than $O(10^{-7})$. Hence, luni-solar gravitational effects were considered negligible.

SOLAR RADIATION PRESSURE

Neglecting the effect of the earth's shadow, the solar radiation pressure disturbing function is [Kaula (3)]

$$\begin{aligned}
R_p = -\frac{3ae}{2} F \left\{ \cos^2 \frac{i}{2} \sin^2 \frac{\epsilon}{2} \cos (\omega + \Omega + \lambda_{\odot}) + \cos^2 \frac{i}{2} \cos^2 \frac{\epsilon}{2} \cos (\omega + \Omega - \lambda_{\odot}) \right. \\
+ \sin^2 \frac{i}{2} \cos^2 \frac{\epsilon}{2} \cos (\omega - \Omega + \lambda_{\odot}) + \sin^2 \frac{i}{2} \sin^2 \frac{\epsilon}{2} \cos (\omega - \Omega - \lambda_{\odot}) \\
\left. - \frac{1}{2} \sin i \sin \epsilon \cos (\omega + \lambda_{\odot}) + \frac{1}{2} \sin i \sin \epsilon \cos (\omega - \lambda_{\odot}) \right\},
\end{aligned}$$

where

i = inclination of satellite's orbit plane to earth's equatorial plane

Ω = longitude of ascending node

ϵ = obliquity of the ecliptic

λ_{\odot} = mean longitude of the sun

F = force constant = $[-4.63 \times 10^{-5} \text{ dynes/cm}^2] \times \frac{A}{m}$

A = presentation area of the satellite $\approx 1.86 \text{ meters}^2$

m = mass of satellite = 145.15 kg.

(As an approximation, the satellite's "presentation area" was assumed to be 1/4 its surface area).

Substituting into the variation equation and again using mean values for the elements, a first approximation yields

$$\begin{aligned}
\delta e = .000040 \cos (\omega - \Omega + \lambda_{\odot}) + .000007 \cos (\omega + \Omega - \lambda_{\odot}) \\
- .000007 \cos (\omega + \lambda_{\odot}) + .000003 \cos (\omega - \lambda_{\odot}).
\end{aligned}$$

Values of e corrected for solar radiation pressure (labeled e_c) are given in Table 1. Figure 2 gives a plot of these values vs. time.

LEAST SQUARES FIT AND HARMONIC ANALYSIS

A least squares fit of the type

$$e = e_0 + A \sin \omega + B \cos 2 \omega$$

was applied to the data in Figure 2 - the resulting equation was

$$e = .002482 + .000161 \sin \omega + .000133 \cos 2 \omega.$$

Figure 3 indicates the closeness of the fit - residuals are given in Table 1. The standard deviation was .000036.

In addition, a 12-point harmonic analysis was performed on the data in Figure 2 for one period of the argument of perigee, beginning 99 days after April 27. The series obtained was

$$\begin{aligned} e = & .002490 + .000149 \sin \omega + .000117 \cos 2 \omega - .000027 \sin 3 \omega \\ & - .000027 \cos 4 \omega - .000011 \sin 5 \omega. \end{aligned}$$

DISCUSSION

It was found that the mean values of the eccentricity of Alouette's orbit listed in Table 1 can be represented by the series

$$\begin{aligned} e = & .002482 + .000161 \sin \omega + .000133 \cos 2 \omega + .000040 \cos (\omega - \Omega + \lambda_\odot) \\ & + .000007 \cos (\omega + \Omega - \lambda_\odot) - .000007 \cos (\omega + \lambda_\odot) + .000003 \cos (\omega - \lambda_\odot). \end{aligned}$$

A more accurate representation for a 140 day period beginning 99 days after April 27 is

$$\begin{aligned}
e = & .002490 + .000149 \sin \omega + .000117 \cos 2 \omega - .000027 \sin 3 \omega - .000027 \cos 4 \omega \\
& - .000011 \sin 5 \omega + .000040 \cos (\omega - \Omega + \lambda_{\odot}) + .000007 \cos (\omega + \Omega - \lambda_{\odot}) \\
& - .000007 \cos (\omega + \lambda_{\odot}) + .000003 \cos (\omega - \lambda_{\odot}).
\end{aligned}$$

In addition, an analysis was made of the argument of perigee. Solar resonances were found for terms in the solar gravitational disturbing function with arguments $2(\lambda_{\odot} - 2\Omega_{\odot} + \Omega)$, $\lambda_{\odot} - \omega_{\odot} - 2\Omega_{\odot} + \Omega$, and $\lambda_{\odot} + \omega_{\odot} - 2\Omega_{\odot} + \Omega$, so the effects of these terms could not be computed by the method used previously. However, corrections were made for luni-solar secular effects and solar radiation pressure. Precession and nutation effects were computed using results obtained by Kozai (4), and were found to be relatively insignificant. Then the secular part of ω was removed by means of a least squares fit of the type

$$\omega = \omega_0 + \dot{\omega} (t - t_0),$$

and the residuals were plotted against time (Figure 4). It is interesting to note that the remaining variation appears similar to that found in the eccentricity (Figure 2), the period again approximating that of the argument of perigee.

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Dr. J. W. Siry, head.

The mean orbital elements were supplied by the Orbit Determination
Section, Burree Richardson, head.

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Table 1
Mean Values of ω and $e - e_c$ Corrected for Solar Radiation
Pressure and Comparison with Least Squares Fit

$t - t_0$	ω	e	e_c	Residuals
0	146.165	.00262	.00264	+2
7	129.203	.00259	.00261	+3
21	92.246	.00248	.00250	-1
28	73.179	.00249	.00251	-2
43	35.239	.00256	.00260	-2
50	19.141	.00253	.00257	-7
57	2.565	.00251	.00256	-6
64	346.096	.00247	.00251	-5
71	328.431	.00239	.00244	-2
78	311.372	.00230	.00235	+1
85	289.665	.00210	.00214	-9
99	246.193	.00220	.00224	-1
106	225.570	.00237	.00241	+5
113	207.545	.00244	.00248	0
127	174.010	.00256	.00260	-3
134	159.097	.00258	.00262	-2
148	126.361	.00255	.00259	+2
162	89.885	.00248	.00250	-1
169	67.343	.00253	.00255	+1
176	51.490	.00254	.00256	-2
183	34.968	.00259	.00260	-2
190	18.434	.00261	.00262	-2
204	346.682	.00255	.00256	0
211	329.351	.00244	.00245	-1
218	311.139	.00237	.00237	+3
225	292.302	.00223	.00222	-2
232	267.748	.00218	.00217	-2
239	246.464	.00230	.00228	+4
246	227.052	.00243	.00241	+6
253	209.096	.00255	.00253	+6
260	191.239	.00264	.00261	+4
267	176.018	.00268	.00265	+3
274	160.457	.00270	.00268	+4
281	145.475	.00273	.00271	+9
288	127.954	.00261	.00259	+1
295	110.000	.00253	.00251	-2
302	90.716	.00253	.00251	0
309	70.641	.00256	.00254	+1
316	53.231	.00260	.00257	0

$a = 1.1589$ earth radii $i = 80.466^\circ$

$t - t_0$ is the number of days since April 27, 1963.

ω is measured in degrees.

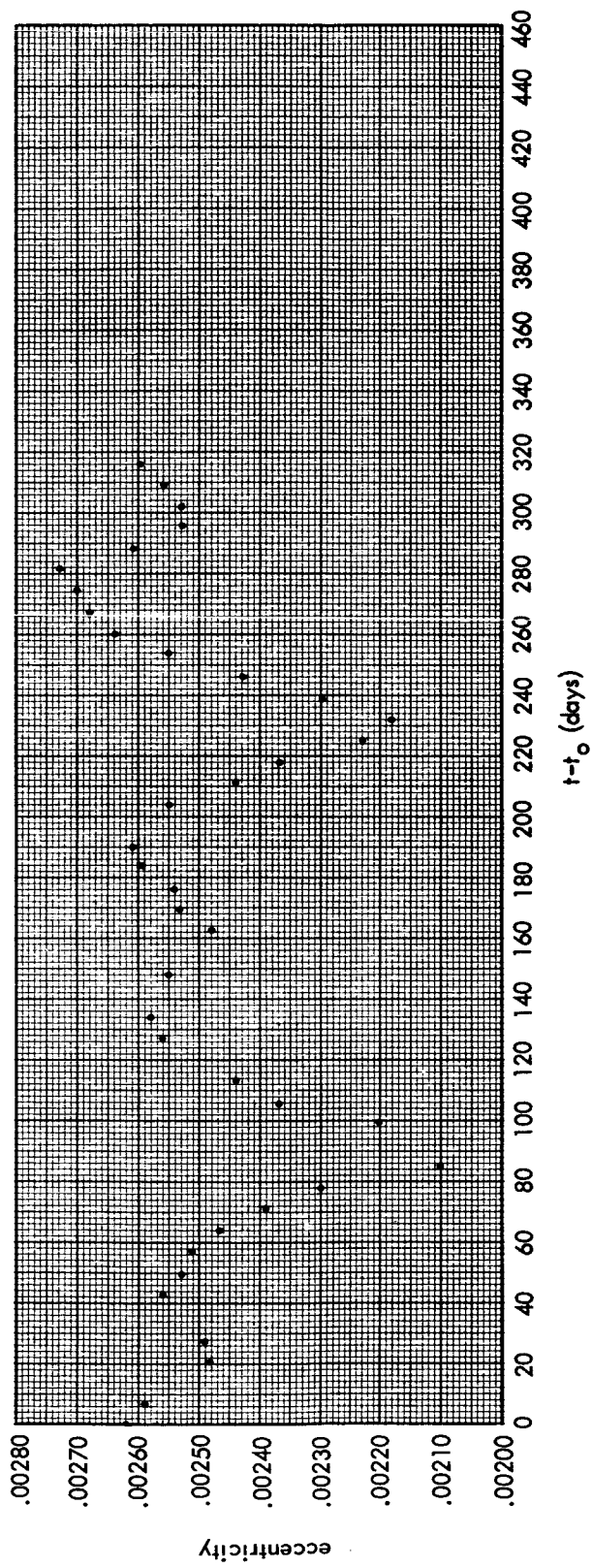


Figure 1. Eccentricity of 1962 β_a (Published values)

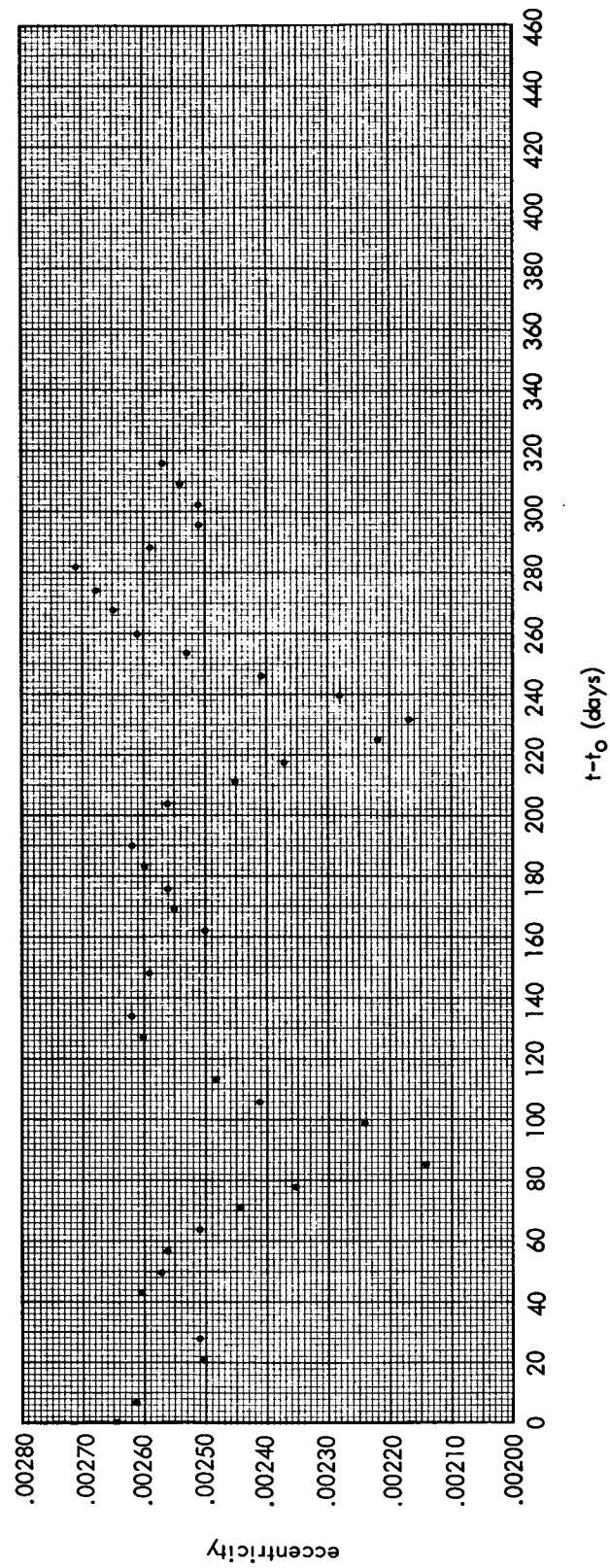


Figure 2. Eccentricity of 1962 β_a (Published values less solar radiation pressure)

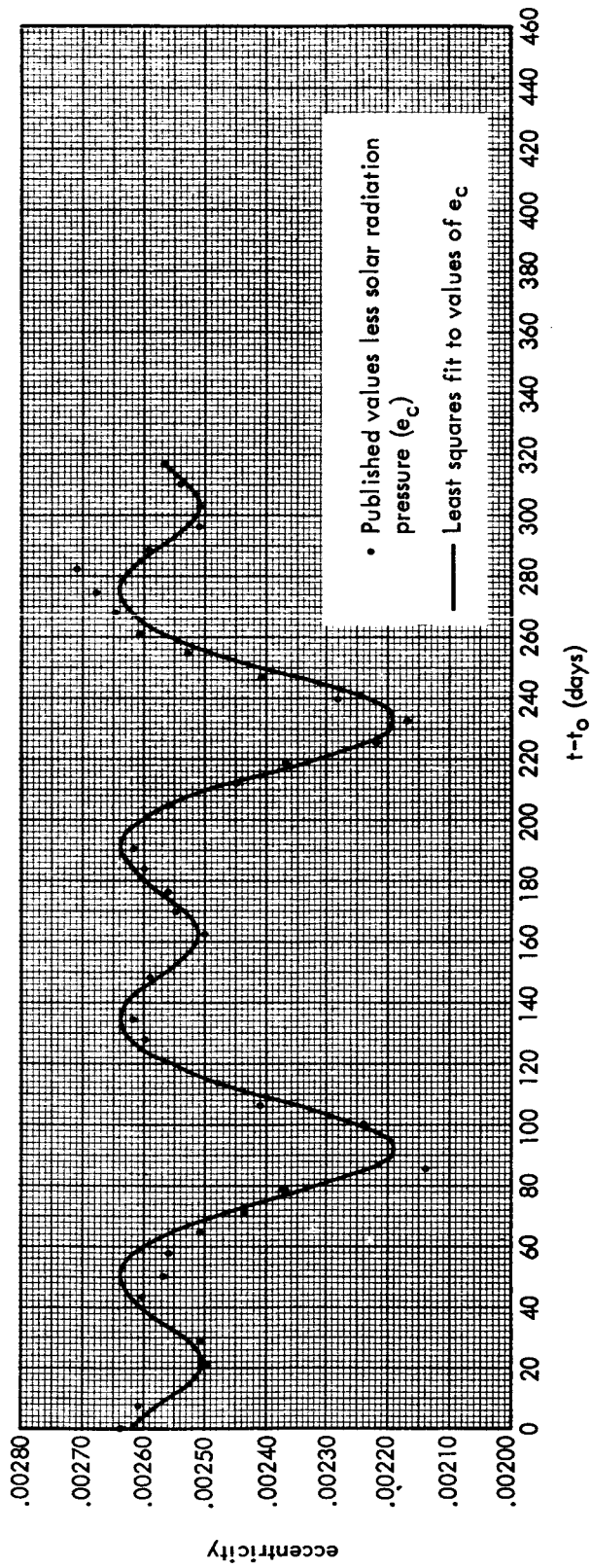


Figure 3. Eccentricity of 1962 β_a

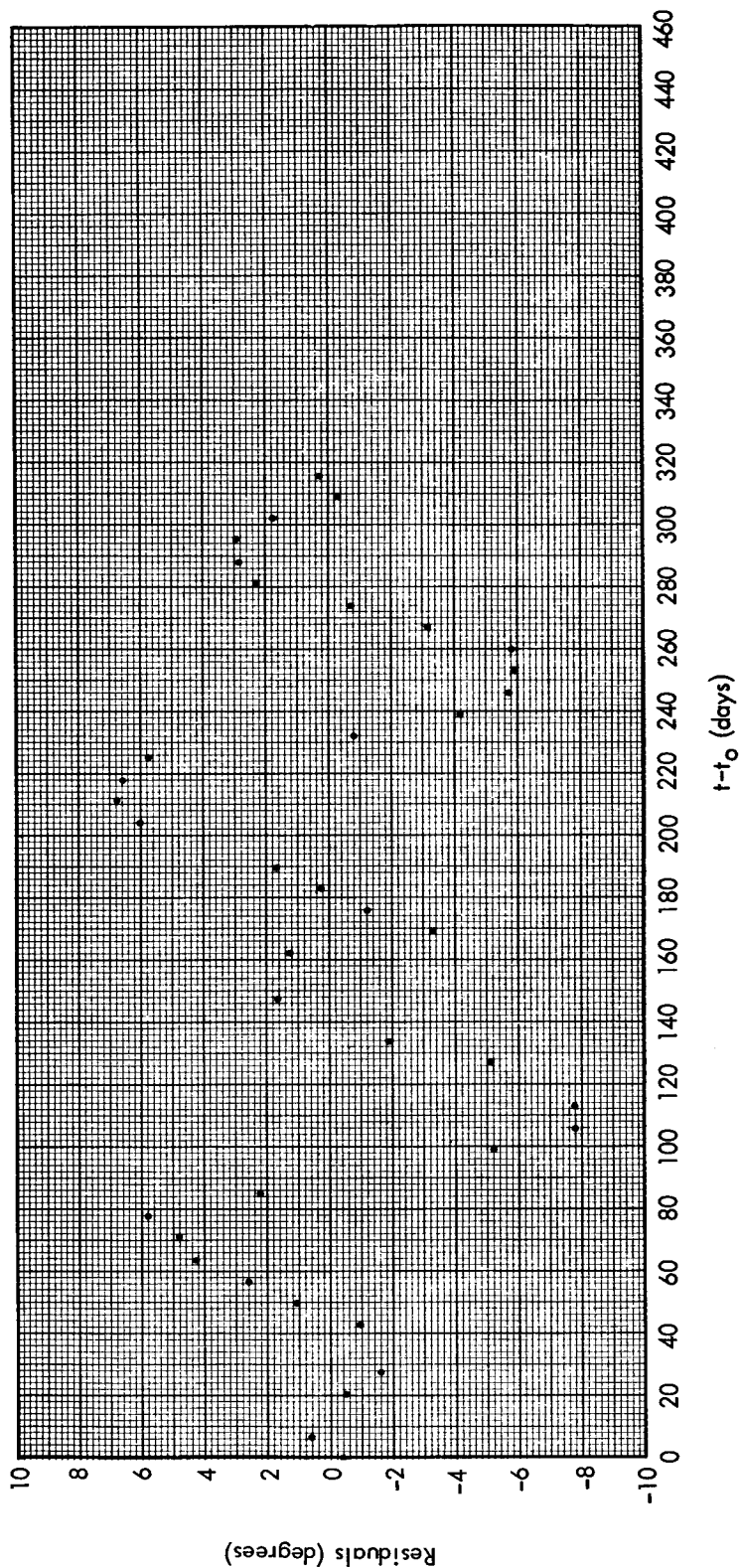


Figure 4. Residuals resulting from least squares fit to argument of perigee (Published values less luni-solar secular effects, solar radiation pressure, and $\omega_0 + \dot{\omega}(t-t_0)$)